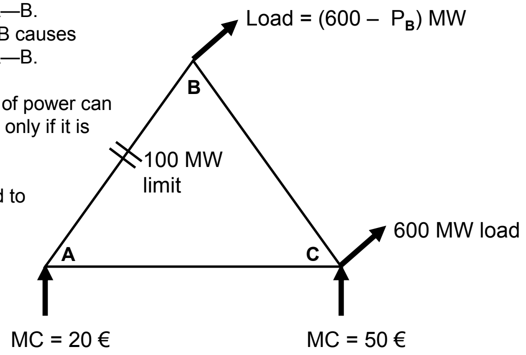
Solution: finding CLPs at A, B, & C

1 MW from A → B causes 2/3 MW on line A—B.
1 MW from C → B causes 1/3 MW on line A—B.

At most 300 MW of power can be sent to B, and only if it is sent from C.

P_B must limit load to 300 MW, so P_B = 300 €.



Checking net benefit

- Has net benefit been maximized?
- 2 MW from C could be replaced by 1 MW from A.
- This would save 80 € of production cost, but it would reduce consumer value by ~2×300 €.
- So net benefit has been maximized.

Finding Price

- Give a trader 1kW of power at the node and see how much money he can make.
- If the power has negative value, he must pay to

What is the price at A, P_A?

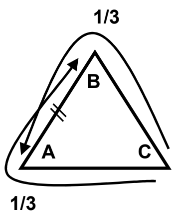
- What is the value of 1 MWh injected at A?
- **If it flows to B** it uses 2/3 MW capacity on line A—B, and this blocks 2 MW from C.
- Consumers lose (at the margin) 300 € / MWh, but 2 × 50 € is saved in production cost, so the value of this MWh = minus 200 €.
- **If it flows to C**, it uses 1/3 MWh capacity A—B, which blocks 1 MWh of energy from C=>B.
- 300 € of value is lost, but 2 MWh is saved at B for a gain of 100 €. Again P_A = –200 €.

What is P_A? (#2)

- If **1 MWh is consumed a A**?
- It can be purchased at A for 20 €.
- But traders always look for the best trade. To find the price at A, we must find the cheapest way to buy power at A.
- It can be purchased from C, and this will reduce congestion on A—B by 1/3 MWh, so one more MW can be sent from C to B.
- This is valuable to consumers at B, so the trader can ask a consumer at B to pay.

What is P_A? (– 200 € again)

- The trade goes to B and asks how much they will pay for 1 MWh. They bid up to 300 €.
- Then the trader goes to C and buys 2 MWh for 100 €.
- Then the trader uses 1 MWh at A while 2 MWh are produced at C.
- 1 MWh flows to A and 1 to B, so congestion is not changed.

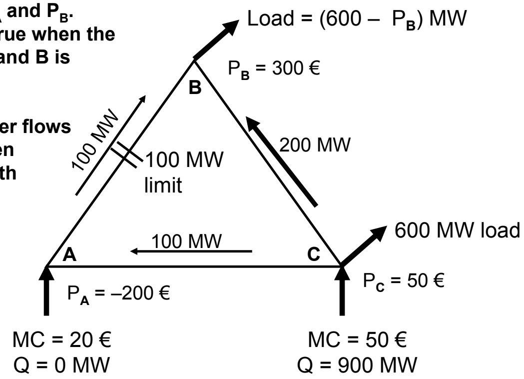


The trader makes 200 € by using 1 MWh at A.

Properties of the Solution

Notice that the P_C is half way between P_A and P_B. This is always true when the line between A and B is congested.

Notice that power flows from C to A, even though it is worth less at A.

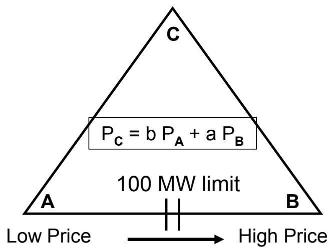


Why is P_C always = $(P_A + P_B)/2$

If A—B is congested towards B, and C needs power, it could buy it all from the high priced node, B. But this is never cheapest.

Instead, buy some from A. But enough must be bought from B to cancel the flow caused by power coming from A.

Suppose 1 MW A→C causes +a MW to flow A→B, and 1 MW B→C causes –b MW A→B.



Then the cheapest way to buy 1 MW at C is to buy b/(a+b) from A and a/(a+b) from B. There will be equal and opposite flows of ab/(a+b) on A—B.

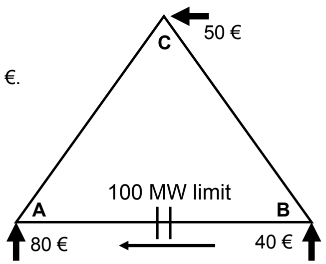
Since a = b = 1/3 in this network, the marginal MW at C must be bought half from A and half from B.

Can we force: $P_C \neq (P_A + P_B)/2$

If C has unlimited generation at a price ≠ $(P_A + P_B)/2$, what happens?

If A—B is not congested, All prices are 40 €.

If B—A is congested,
(1) Suppose the load is all at **A**.
A trader at C could offer to sell 1 MW to someone at **B** for 20 €, because then he could sell 1 MW to **A** for 80 €. Revenue = cost = 100 €.



(2) Suppose the load is all at **C**. A Trader at **B** cannot sell power to **C** unless she buys an equal amount of power from **A** to send to **C**. But 1 MW at **A** costs $80 €, so he can only afford to pay 20 € at **B**. Before the line is congested, the price at B is 40 €, but after congestion, it drops to 20 €.

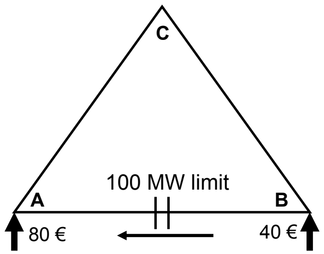
Either way the price at B, with B→A congested, is 20 €, not 40 €.

Another way to understand CLPs

If line A—B were expanded by 1 MW,
1.5 more MW could flow from B to A,
1 through A—B and one through node C.

So the "expansion value" (EV) of the line is 60 € / MWh.
That is the price difference times the total extra power flow.

The nodal price at any node C, can be found from the price at B, and the fraction of power that flows on B→A when power is sent from B to C.
That fraction is called a "distribution factor," DF_C.



$EV_{BA} = (P_A - P_B) / DF_A$

$P_C = P_A + EV_{BA} * DF_C$

In this case $P_C = 40 + 60 \times (1/3) = 60$

Distribution factors are constant

- Because distribution factors are constant, the expansion value can be found from the price difference $P_A - P_B$.
- Knowing this EV_{BA} and the price at B, and all the other power factors, every nodal price can be found.
- If there are N congested lines then all nodal prices can be found from P_B and the EVs of the N congested lines, and the distribution factors.
- No other economic information is needed.

Distribution factor simplification

- Distribution factors and "expansion values" and one nodal price make it easy to find all the other nodal prices.
- But it is still just as hard to find all the "expansion values."
- So this method helps us understand nodal prices, but it is not a shortcut for central computation.